DIOPHANTINE ANALYSIS.

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SOLUTIONS TO PROBLEMS.

6. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Find three whole numbers the sum of any two of which is a cube.

II. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x, y, and z=the numbers. Then $x+y=a^3$, $x+z=b^3$. and $y+z=c^3$; in which $c>b>a$. Hence $x+y+z=\dfrac{a^3+b^3+c^3}{2}$.

$$\therefore\ x=\frac{a^3+b^3+c^3}{2}-c^3;\ y=\frac{a^3+b^3+c^3}{2}-b^3;\ z=\frac{a^3+b^3+c^3}{2}-a^3.$$

In order that each of the numbers be integral, *either* each of the cubes must be even, *or* two of the cubes must be odd and one even. Also, in order that each of the numbers be positive, $a^3+b^3>c^3$.

The simple rule for finding the three numbers is as follows: Take three cubes fulfilling the above two conditions, and from half their sum subtract each cube separately; the remainders will be the three numbers required.

It is evident that many sets of numbers be obtained. We will illustrate by the following consecutive cubes: 1, (2) 8, (3) 27, (4) 64, (5) 125, (6) 216, (7) 343, (8) 512, (9) 729, (10) 1000, (11) 1331, (12) 1728, (13) 2197, (14) 2744, (15) 3375, (16) 4096, (17) 4913, (18) 5832.

The first three cubes answering the above conditions are 343, 512, and 729. $\frac{1}{2}$ their sum is 792; $x=792-729=63$; $y=792-512=286$; $z=792-343=449$. The next three cubes are $(7)^3$, $(9)^3$, and $(10)^3$. The first three *even* cubes are $(12)^3$, $(14)^3$, and $(19)^3$.

9. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

It is required to find three numbers the sum of whose 4th powers is a square.

I. Solution by R. J. ADCOCK, Larchland, Warren County, Illinois.

If $u=x^2+y^2-z^2$, then $u^2=x^4+y^4+z^4+2x^2y^2-2x^2z^2-2y^2z^2$, in which if $2x^2y^2-2x^2z^2-2y^2z^2=0$, $z^2=\dfrac{x^2y^2}{x^2+y^2}$, and $u=x^2+y^2-\dfrac{x^2y^2}{x^2+y^2}$

$$=\frac{x^4+y^4+x^2y^2}{x^2+y^2};\ u^2=x^4+y^4+z^4=x^4+y^4+\frac{x^4y^4}{(x^2+y^2)^2}=\left(\frac{x^4+y^4+x^2y^2}{x^2+y^2}\right)^2.$$

$\therefore\ x^4(x^2+y^2)^2+y^4(x^2+y^2)^2+x^4y^4=(x^4+y^4+x^2y^2)^2$, which is a general equa- for the sum of the 4th powers of three quantities=a square, when x^2+y^2=a tion square. Making $x=3, y=4$, $x^2+y^2=5^2$, and $3^4\times5^4+4^4\times5^4+3^4\times4^4=15^4+20^4+12^4=(3^4+4^4+12^4)^2=481^2=231361$.